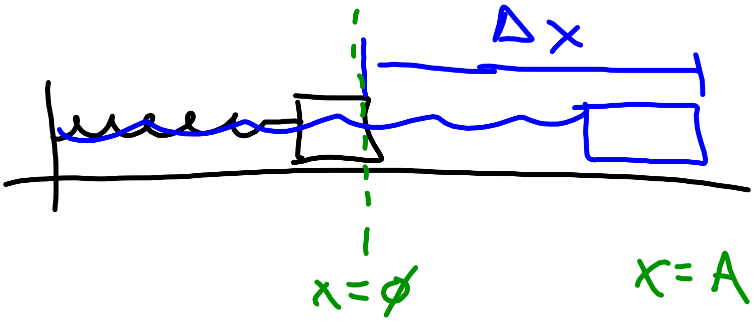
Simple Harmonic Motion



$$F = -kx$$

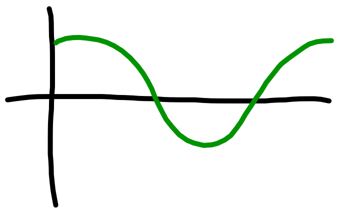
$$v = \frac{dx}{dt}$$

$$ma = -kx$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$$m\frac{d^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$



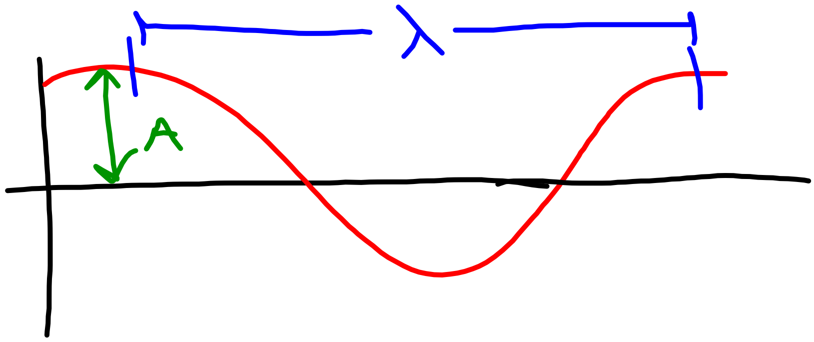
$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

Must have function that satisfies both sides

$x = x^3$ Doesn't work!

$x = \sin x$

$$-\sin x = -\frac{k}{m}\sin x$$



T → time for 1 full cycle

f → cycles per second

$$T = \frac{1}{f}$$

$$x(t) = A \cos(\omega t + \varphi)$$

Amplitude (A)

angular frequency (ω) (1/s)

phase shift (φ) (degrees or radians)

$$\frac{dx}{dt} = v(t) = -A\omega \sin(\omega t + \varphi)$$

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = a(t) = -A\omega^2 \cos(\omega t + \varphi)$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

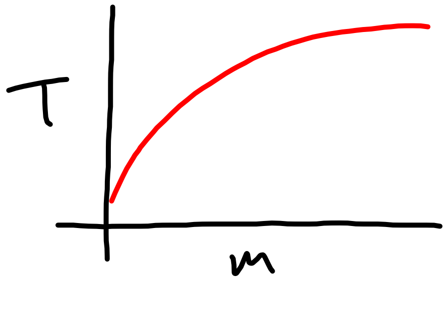
$$+\cancel{A}\omega^2 \cancel{\cos(\omega t + \varphi)} = +\frac{k}{m}\cancel{A\cos(\omega t + \varphi)}$$

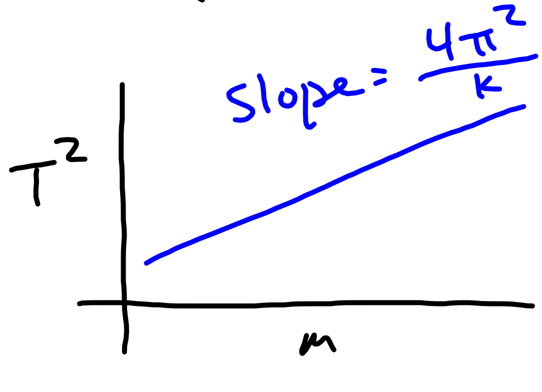
$$\omega^2 = \frac{k}{m}$$

$$\omega = 2\pi f = \sqrt{\frac{k}{m}} \qquad f = \frac{1}{T}$$

$$\frac{2\pi}{T} = \sqrt{\frac{k}{m}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$





$$e^{i\theta} = \cos\theta + i\sin\theta$$

Euler's equation

$x(t) = A\cos(\omega t + \varphi)$

maximum at $\emptyset°/180°$

$x = \pm A$ $\quad \emptyset/\pi$

minimum at $90°/270°$

$x = \emptyset$ $\quad \pi/2 \,/\, 3\pi/2$

$v(t) = -A\omega\sin(\omega t + \varphi)$

maximum at $90°/270°$

$v = \pm A\omega$ $\quad \pi/2 \,/\, 3\pi/2$

minimum at $\emptyset°/180°$

$\emptyset/\pi$

$v = \emptyset$

$a(t) = -A\omega^2\cos(\omega t + \varphi)$

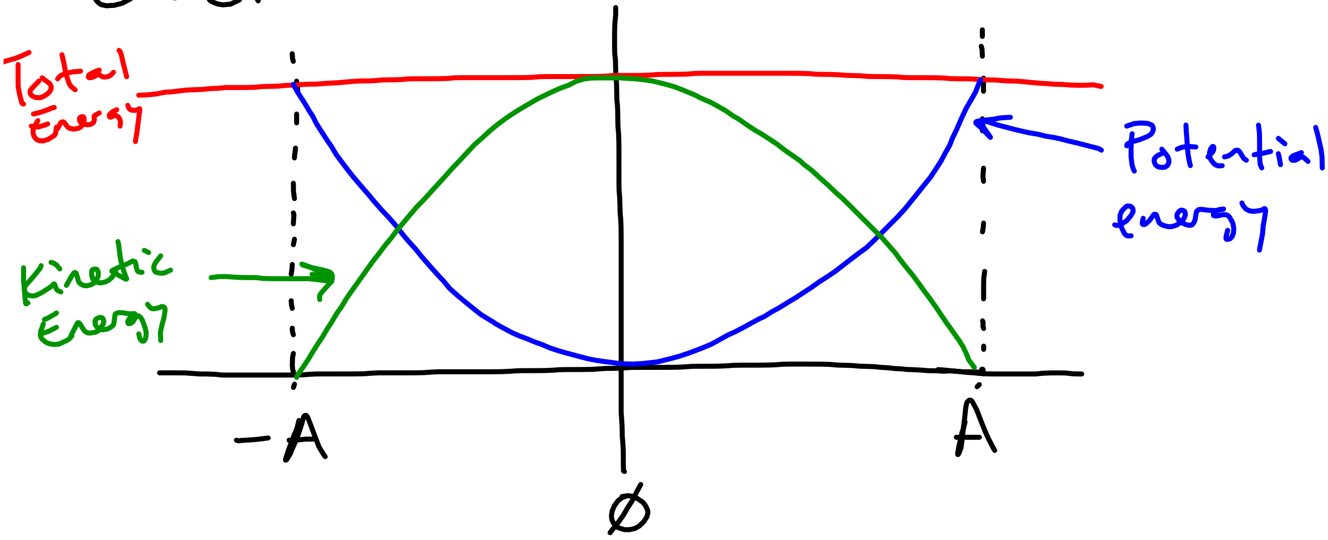
maximum at $\emptyset°/180°$

$a = \pm A\omega^2$ $\quad \emptyset/\pi$

minimum at $90°/270°$

$a = \emptyset$ $\quad \pi/2 \,/\, 3\pi/2$

ENERGY



$$E = U + K$$

$$E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

$$E = \frac{1}{2}k\left[A\cos(\omega t+\varphi)\right]^2 + \frac{1}{2}m\left[-A\omega\sin(\omega t+\varphi)\right]^2$$

$$= \frac{1}{2}kA^2\cos^2(\omega t+\varphi) + \frac{1}{2}mA^2\omega^2\sin^2(\omega t+\varphi)$$

$$\omega^2 = \frac{k}{m}$$

$$= \frac{1}{2}kA^2\cos^2(\omega t+\varphi) + \frac{1}{2}kA^2\sin^2(\omega t+\varphi)$$

$$= \frac{1}{2}kA^2\left[\cos^2(\omega t+\varphi) + \sin^2(\omega t+\varphi)\right]$$

$\cos^2(\ ) + \sin^2(\ ) = 1$!

$$E = \frac{1}{2}kA^2$$

Unless told otherwise, assume

$\varphi = \emptyset$!

PRACTICE

1) a. $T = \frac{10\ s}{15} = \frac{2}{3}\ s$

b. $v_{max} = -A\omega \cancel{\sin(\omega t + \varphi)} \rightarrow 1$

$= -A\omega \qquad \omega = \frac{2\pi}{T}$

$= -A\left(\frac{2\pi}{T}\right)$

$= -(0.2\ m)\left(\frac{2\pi}{2/3\ s}\right)$

$= \pm 1.88\ m/s$